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STROP:
A STRATEGIC PLANNING MODEL

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PREFACE

This Memorandum describes one of the models developed in a continuing project at RAND concerned with strategic planning techniques. The particular model described is designed to be one of an integrated family of computer models for generating strategic war plans and evaluating force structures.

SUMMARY

STROP is a highly aggregated central nuclear war game, coded for a high-speed computer, which evaluates a pair of Red and Blue allocations of missiles and bombers to some combination of four target systems: missile sites, bomber fields, bomber defenses, and value targets. The routine will evaluate one pair of Red and Blue allocations in about 1/50 of a second. The routine can be used to generate and survey a large sample of Red and Blue allocations or to evaluate specific allocations selected by the analyst. STROP is designed as one element of a family of central nuclear war models concerned with the generation of strategic war plans and the evaluation of strategic force structures.

ACKNOWLEDGMENTS

David Langfield has contributed extensively both in the design of the model and in programming it for the 7044. Margaret Ryan assisted in several contributing substudies, including the coding and analysis of a preliminary form of the missile interaction model.

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STROP: A STRATEGIC PLANNING MODEL

1. INTRODUCTION

For many problems of interest to the military, it seems desirable to have a small, rapidly computable model of central nuclear war that can be used to explore in a quick fashion a wide range of parameters. The need is particularly strong in the area of strategic planning where the number of possible plans is very large and where the effectiveness of a plan depends in a crucial manner on the plan adopted by the enemy [1]. The model described in the present paper, called STROP, is a highly aggregated central nuclear war game coded for an IBM 7040-44. It can evaluate the outcome of a nuclear exchange in about 1/50 of a second. Thus it is feasible to examine a large sample of potential conflicts.*

STROP is basically a static model; however, time enters in determining the average number of bombers or missiles that will be at risk during enemy attack (empty-hole computation), in assessing the effect of warning and relative execute times, and in assessing the effect of rate of fire. This last can also be used to take into account the results of withholding portions of the forces for later attack. The model includes missiles, bombers, bomber area defense, local defenses, antimissile defenses, value targets, and fallout casualties. The effect on bomber attrition of attacking area defenses with missiles and bombers is assessed. The outcome of the nuclear exchange can be evaluated assuming either that the attacks

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*Because the model necessarily contains major simplifications, it should be supplemented by more detailed examination of those cases that appear significant. STROP is designed to fit into a family of models that can take specific STROP runs and "unpack" them into more complete simulations of nuclear wars. These models are briefly described in refs. 3-5.

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on both sides are completely preplanned, or that one or both sides have a retargeting capability to compensate for losses to enemy fire.

STROP is an expected value model. At present there is no simple technique for determining the variance due to chance. From the standpoint of an integrated family of models this is not a serious drawback. STROP can be supplemented by more extensive models—e.g., of the monte-carlo type—which can explore the variation due to chance in those cases which warrant further investigation.

In STROP, a strategic plan consists of an allocation of offensive weapons to targets—i.e., a statement of what fraction of bombers and missiles will attack each of the target systems. Several modes of exploration of the allocation possibilities have been included. (1) The model can be used to evaluate a specific pair of allocations, one for Red and one for Blue, as input by the analyst. (2) The model will automatically generate a large sample of allocations on both sides and compute the outcomes of each pair of allocations. (3) The matrix of outcomes of these sample pairs can be printed out for study. (4) The routine can scan the matrix for dominance on each side and print out the reduced matrix of undominated allocations. (5) A refined form of payoff based on the assumption of increasing concern (discussed more fully below) can be defined and the reduced matrix further decreased by eliminating allocations dominated with respect to this payoff. (6) A submatrix (e.g., the undominated set) can be "blown-up" so that a denser sample is taken in the subregion.

There is no presently known form of "solution" to nonzero sum games that is satisfactory for military analyses. The concern with STROP has been to furnish the analyst with a variety of ways of working with the model so that he can select one or more that is appropriate for his problem.

In addition to exploring strategic plans, STROP can be used to examine force structures in a two-sided context. It has an elementary cost model where the numbers and capabilities of new weapons are a (nonlinear) function of the amount of money allocated to each kind of force element. The model will select a sample of force structures for each side at a fixed (input) budget and compute the outcome of a war fought with each sample pair.

The problem of evaluating force structures is much more difficult than evaluating strategic plans with fixed forces. The effectiveness of a force structure is a function of both the enemy forces and the force employment on both sides. Thus for each pair of forces, it is necessary to determine preferred strategic plans on each side. If a sample of strategies like that used in the test runs for STROP (168 x 168) were run for each pair in a sample of force structures of about the same size, a continuous run lasting nearly a year would be required!

Several ways of reducing the running time have been developed. A suboptimizing algorithm for allocating bombers between attacks on bomber defenses and attacks on value targets has been added. Using this algorithm, a reduction in running time by a factor of about 10 is obtained. Simple dominance, when the increasing concern payoff is used, appears to lead to a unique undominated pair of allocations. More direct ways of finding this pair can be used, and a routine has been coded to search for the "max-max" point which, in almost all cases, does not require examining every strategy pair. Finally, a simplified form of the increasing concern payoff can be defined which shortens the computation. An additional decrease in running time by a factor of about 3 is achieved by these two measures.

With the preceding simplifications, the running time of a strategy exploration in STROP comparable to the 168 x 168 previously mentioned, is reduced to about ½ minute.

This is still somewhat longer than might be preferred, considering the level of aggregation of the model, but is probably tolerable for many force structures investigations.

2. ELEMENTS OF THE MODEL

The basic elements of STROP are indicated in Table 1. The symbols represent the number of elements in each category. The expression for missile sites is in parenthesis since there is no separate tabulation of missiles and missile sites, the distinction being implemented by defining an average occupancy of missiles sites for purposes of computing losses to enemy fire. In the present model, Local Defenses and AICBM's are not considered as targets, but are expressed in terms of the number of defended cities.

Bombers and missiles each have a basic effectiveness in attack on those elements that are targets, as expressed in Table 2. This basic effectiveness includes the CEP of the weapon, yield of warhead, height of burst, warhead reliability, and vulnerability of the target. Depending on the interest of the analyst, it can also include any of several abort factors (e.g., in-flight abort of bombers).

The basic effectiveness, then, is the probability that a weapon which survives the enemy counterforce attack will destroy a target of the given type. This is exclusive of attrition enroute to target.

"Cities" are interpreted as value target <u>units</u>, i.e., each could be interpreted as separate DGZ's. The measure for these units could be population, industrial output, or any other value parameter. A large complex target would be expressed as a number of these value target units.

In addition to the offensive effectiveness for weapons, the model includes a basic kill probability for area defenses against bombers, for local defenses against bombers, and for AICBM's against missiles. These are discussed more fully in the section on attrition. Fallout effects are expressed in terms of a table which relates fallout casualties to type of weapon and type of target.

Table 1

STROP — ELEMENTS

	Blue	Red
Missiles	M_B	M_R
Bombers	B_B	B_R
Missile sites	(O_{MB})	(O_{MR})
Bomber bases	F_B	F_R
Area defenses	D_B	D_R
Local defenses	(L_B)	(L_R)
Missile defenses	(A_B)	(A_R)
Cities (value targets)	C_B	C_R

Table 2

STROP — BASIC EFFECTIVENESS

	Blue	Red
Missile effectiveness		
Missile sites	u_{MB}	u_{MR}
Bomber fields	u_{FB}	u_{FR}
Area defenses	u_{DB}	u_{DR}
Cities	u_{CB}	u_{CR}
Bomber effectiveness		
Area defenses	q_{DB}	q_{DR}
Cities	q_{CB}	q_{CR}

A plan, for STROP, consists in a statement of the proportion of each offensive weapon that will attack each target system as indicated in Table 3. Weapons can also be allocated to SAVE. There is no formal requirement that the allocations add up to 1; they must, of course, add up to less than or equal to one. If the sum is less than one, the surplus is not automatically allocated to SAVE. The SAVE category is reserved for forces deliberately withheld.

Table 3

STROP — ALLOCATION

	Blue	Red
Missiles allocated to		
Missile sites	x_{MB}	x_{MR}
Bomber fields	x_{BB}	x_{BR}
Area defenses	x_{DB}	x_{DR}
Cities	x_{CB}	x_{CR}
SAVE	x_{SB}	x_{SR}
Bombers allocated to		
Area defenses	y_{MB}	y_{MR}
Cities	y_{CB}	y_{CR}
SAVE	y_{SB}	y_{SR}

3. ALLOCATION GENERATION

The allocation generator prepares a list of potential allocations for each side. The planner specifies the allocation "step"—that is the increment by which the fraction allocated to a given target system will be varied. The generator allocates missiles and bombers separately. In the case of missiles the first allocation is 100% to missile sites, the second 100% to bomber bases, and so on. After allocating 100% of missiles to each of the target systems, the generator then "steps down" one increment and, e.g., if the input increment is 20%, allocates 80% to missile sites 20% to bomber bases, 80% to missile sites and 20% to fighter bases, etc. In this fashion it generates all combinations of allocation of multiples of the increment which add up to one. For example, if the increment is 20%, there are 56 combinations for missiles. The procedure is similar for bombers, except that the present routine generates allocations to bomber defenses and value targets only. In addition there is a restriction that no more than 50% of the remaining bombers will be allocated to bomber area defenses. If the allocation step is 20% there will be only 3 allocations for bombers. With the 56 for missiles, this produces 168 combinations over all. The generator can be requested to count the number of combinations or to prepare the list for use by STROP. The counting option is included to enable the analyst to decide whether the number is acceptable in the light of memory space and computation time available.

In addition the generator will accept a list of specific allocation options and generate a sample from these. For example, the analyst may wish to keep the fraction of missiles allocated to bomber fields and to missiles fixed and vary the fractions allocated to bomber defenses and to value targets.

4. OCCUPANCY

The allocation generator computes a table of occupancy numbers for evaluation of countermissile and counterbomber attacks. Occupancy can be defined as the average number of missiles or bombers that remain unlaunched when enemy missiles arrive. It is a function of the warning times for each side, their generation rates, as well as the size of the enemy counterforce attack which (in STROP, at least) determines how long the attack will last. Occupancy numbers for missile sites and bomber fields are precomputed, tabled in terms of the duration of the counterforce attack, and stored for later use.

Time enters into STROP only through the occupancy computation. The basic parameters are (a) generation rates, (b) warning time, and (c) execute time. Generation rates for both bombers and missiles are described by curves of the form illustrated in Fig. 1.

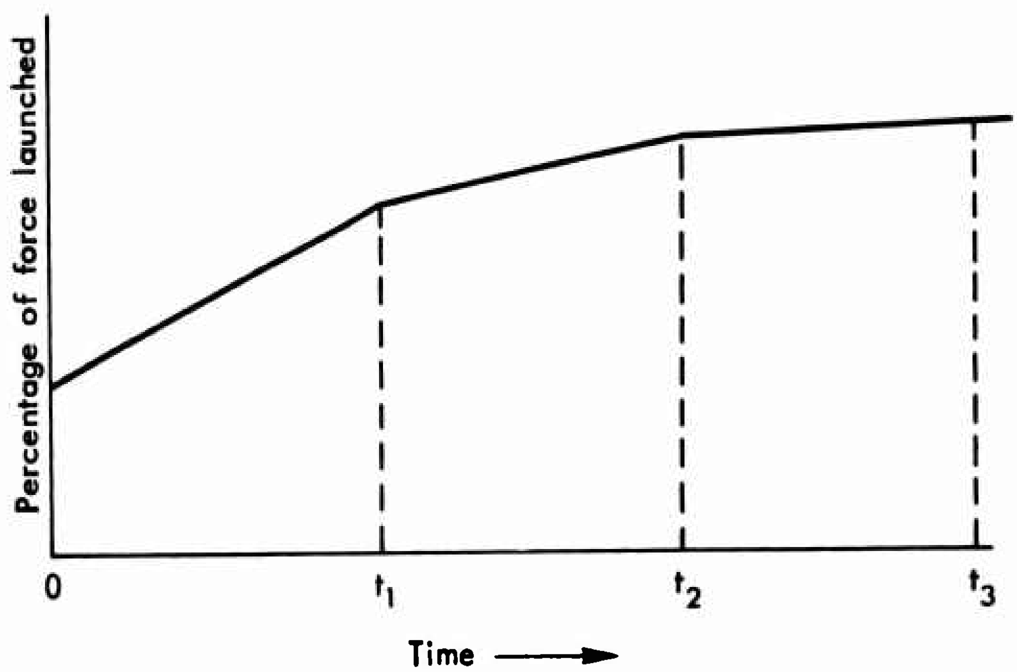

Fig. 1—Generation curve

Internally the generation curves are expressed as three line segments. Strictly speaking, the curves are launch schedules, in the sense that the model assumes that weapons will be launched at the specified times. This enables the generation curves to be used to indicate withheld forces. For example, a curve of the form shown in Fig. 2 would

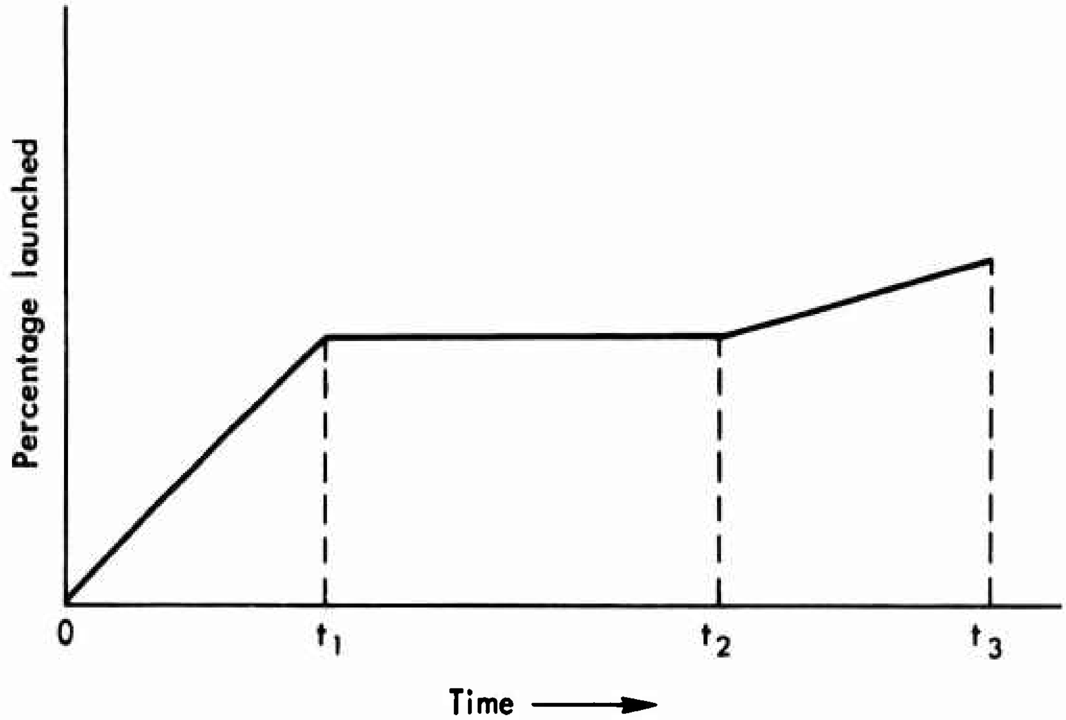

Fig. 2—Generation curve, withholding case

indicate an initial exchange up to t_1; a period of withholding, between t_1 and t_2; and a subsequent follow-on attack starting at t_2. The precise way in which this form of the generation curves can be used to evaluate withholding will be discussed in the section on criteria.

Generation does not have to reach 100% by the last time, t_3, considered in the interaction. Thus, another form of withholding can be expressed simply by stopping generation before 100%.

Weapons generated at 0 can be considered as on airborne alert for bombers, or as some absolutely invulnerable portion of missiles. Weapons on ground alert launch at some finite rate, although this can be as rapid as the planner wishes.

The effect of warning is to transform the generation curves to increase the number of weapons on alert. For example, Fig. 3 illustrates the effect of a 35-minute

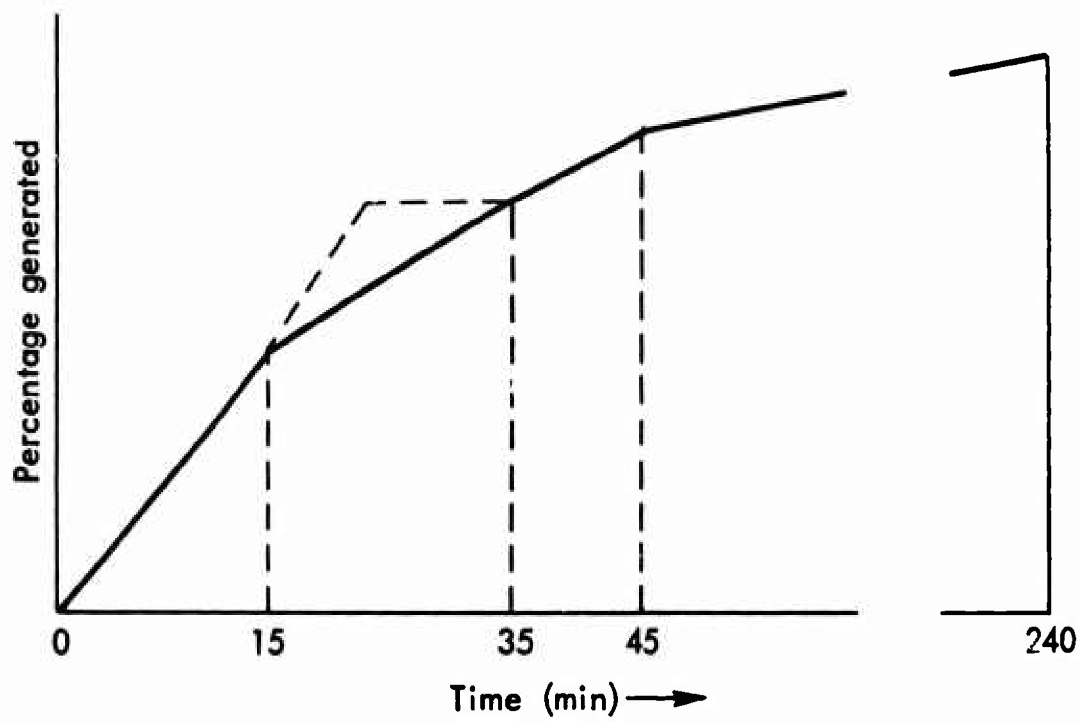

Fig. 3—Effect of warning

warning. All weapons that would have been generated up to 35 minutes are launched at the same rate as alert weapons. This produces the dashed curve in the figure.

Execute time determines the time at which weapons begin launch. Separate times can be input for missiles and bombers, allowing the simulation of bombers launched on positive control.

The occupancy computation is illustrated in Fig. 4 The curve above the center line represents the arrival of attacking missiles as a function of time. It is simply the generation curve of the attacker translated by missile flight time and execute time. The lower curve is the generation curve of the attacked side translated by execute time. The curve will be for missiles or bombers depending on which occupancy is being computed.

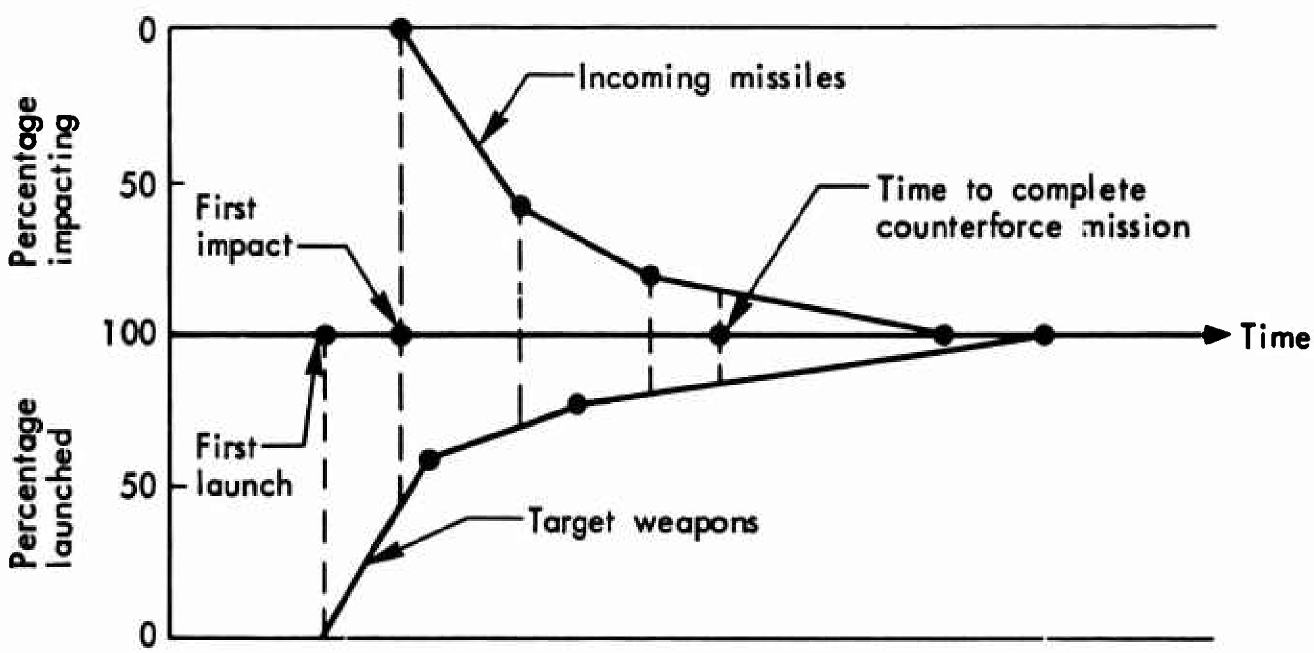

Fig. 4—Occupancy computation

The assumption is made that, for the first exchange, any counterforce attacks will precede any attacks on value targets. This assumption is based on the consideration that counterforce targets are fleeting and therefore should be attacked first. It is also assumed that, if both bomber fields and missiles sites are to be attacked, these two attacks will proceed simultaneously. Depending on the relative allocation of missiles to counterforce and other targets, and the generation rate, there will be a time at which the counterforce mission will be completed, as noted in Fig. 4.

The routine does an approximate numerical integration of the product of the two curves between first impact of incoming missiles and time to complete the counterforce mission to obtain the average number of weapons remaining unlaunched at impact. For fixed generation curves and execute times, the occupancy depends only on the time to complete the counterforce mission. Thus, in the strategy generator, an occupancy table is prepared for each side, as a function of the percentage of forces allocated to counterforce on the other side.

5. MISSILE INTERACTION

The basic missile interaction model is given in Eqs. (5.1) and (5.2), where $\bar{M}_B$ and $\bar{M}_R$ are the number of Blue and Red missiles remaining after the interchange:

$$(5.1) \qquad \bar{M}_B = M_B - O_{MB} \min[M_B, \bar{M}_R \cdot x_{MR} \cdot \bar{u}_{MR}],$$

$$(5.2) \qquad \bar{M}_R = M_R - O_{MR} \min[M_R, \bar{M}_B \cdot x_{MB} \cdot \bar{u}_{MB}].$$

O_{MB} and O_{MR} are the Blue and Red missile occupancies described in the preceding section. $\bar{u}_{MB}$ and $\bar{u}_{MR}$ are the average effectiveness of Blue and Red missiles on missiles. They depend on the single-shot effectiveness u, on the number of warheads per target, and on ground survival in the case of preplanned attacks. This dependence is expressed in Eqs. (5.3) and (5.4), where Q is the fraction surviving and N is the number of warheads per target:

$$(5.3) \qquad \bar{u} = \frac{1 - [1 - Qu]^N}{QN},$$

$$(5.4) \qquad \bar{u} = \frac{1 - [1 - u]^N}{N}.$$

Equation (5.3) is the appropriate expression for preplanned attack, (5.4) for an attack with retargeting taking into account losses to enemy fire. For the pretargeting case, the number of warheads per target is obtained by dividing the number of warheads allocated to missiles (number of missiles allocated times the number of warheads per missile) by the number of missile targets. For the retargeting case, N is the <u>surviving</u> number of warheads allocated to missiles divided by the number of targets. That is,

$$(5.5)\qquad N = \max\left[\frac{M_B \cdot x_{MB}}{M_R}, 1\right] \quad \text{pretargeting}\ ,$$

$$(5.6)\qquad N = \max\left[\frac{\bar{M}_B \cdot x_{MB}}{M_R}, 1\right] \quad \text{retargeting}\ .$$

(Equations identical in form to (5.3) and (5.4) are used to determine the average effectiveness of both missiles and bombers against all other target systems. However, the method of computing N depends on the target system.)

Equations (5.1) and (5.2) are a pair of simultaneous equations essentially in two unknowns. They could be solved directly if they did not contain the min operation and if the $\bar{u}$ did not depend on survival. The compute routine solves these equations by iteration, starting with $Q_{MB} = Q_{MR} = 1$. The approximate solution is iterated until the last difference is less than an input number or until an input number of iteration steps have occurred.*

Bombers surviving missile attack, $\bar{B}_B$ and $\bar{B}_R$, are computed according to Eqs. (5.7) and (5.8). In the present version of STROP, bombers do not attack missile sites, nor enemy bomber bases,** thus there is no need to iterate (5.7) and (5.8); they can be computed directly:

$$(5.7)\qquad \bar{B}_B = B_B - O_{BB} \min[F_B, \bar{M}_R \cdot x_{BR} \cdot \bar{u}_{BR}],$$

$$(5.8)\qquad \bar{B}_R = B_R - O_{BR} \min[F_R, \bar{M}_B \cdot x_{BB} \cdot \bar{u}_{BB}].$$

O_{BB} and O_{BR} are the Blue and Red bomber occupancies. The other factors are defined in Tables 1 – 3.

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*In all test runs the approximation has converged rapidly. Three or four iteration steps have been sufficient to determine $\bar{M}$ to within 1 percent.

**In a later version, this limitation will be relaxed.

6. ATTRITION

STROP computes bomber losses to enemy area defenses and enemy local defenses and missile losses to AICBM's. Bomber area defenses are one of the potential targets, and STROP evaluates the degradation of area defense effectiveness due to missile or bomber attack. Local defenses (SAM's) and AICBM's are not subject to attack in the present version.

The basic input to the attrition computation is the kill probability against bombers of undegraded defenses. This kill probability is degraded by the fraction of defenses remaining in the case of an attack on defenses, as indicated in Eqs. (6.1) – (6.4):

$$(6.1) \qquad \bar{D}_R = D_R - \min[D_R, \bar{M}_B \cdot x_{DB} \cdot \bar{u}_{DB}] ,$$

$$(6.2) \qquad \bar{P}_R = \frac{\bar{D}_R}{D_R} P_R ,$$

$$(6.3) \qquad \bar{\bar{D}}_R = \bar{D}_R - \min[\bar{D}_R, \bar{B}_B \cdot y_{DB} \cdot \bar{q}_{DB}(1 - \bar{P}_R)] ,$$

$$(6.4) \qquad \bar{\bar{P}}_R = \frac{\bar{\bar{D}}_R}{D_R} P_R .$$

Equation (6.1) defines defenses surviving missile attack, Eq. (6.3) defenses surviving bomber attack. Bombers attacking defenses meet area defenses degraded by missiles only; bombers attacking value targets meet defenses degraded by both missiles and bombers. Thus, Blue bombers allocated to value targets have a survival expressed by $(1 - \bar{\bar{P}}_R)$. Bombs surviving involve an additional factor expressing the probability that, if a bomber is shot down, it has already dropped one or more bombs. Bomb survival is thus $(1 - w\bar{\bar{P}}_R)$, where w is the probability that a bomb was not released prior to destruction of the aircraft.

Since local defenses are not targets in the present version of STROP, attrition to local defenses is expressed as a simple kill probability at defended targets. Only value targets are defended by local defenses. The level of local defense is expressed by the number of defended targets.

The AICBM model is similar to the local defense model with the one addition already mentioned: warheads are allocated to defended and undefended targets so as to equalize the kill probability on the two kinds of targets, with the restriction that no less than one warhead is allocated per target. It is assumed that only value targets are defended by AICBM's, and that AICBM's are not themselves targets.

Targets that have both local defenses against bombers and AICBM defenses are attacked before those that are defended only by AICBM's.

7. VALUE-TARGET DAMAGE

Damage to value targets is computed in a manner similar to that for other targets with some modifications required by the fact that value targets may be defended by SAM's or AICBM's. In the absence of local defenses, damage to value targets is simply

$$(7.1) \qquad D_R = \bar{M}_B \cdot x_{CB} \cdot \bar{u}_{CB}$$

for missiles and

$$(7.2) \qquad D_R = \bar{B}_B \cdot y_{CB} \cdot \bar{q}_{CB} \cdot (1 - w\bar{\bar{P}}_R)$$

for bombers.

In the case of bombers, $\bar{q}$ contains area attrition and the warhead kill factor, i.e.,

$$(7.3) \qquad \bar{q}_{CB} = \frac{1 - (1 - (1 - w\bar{\bar{P}}_R) \cdot Q_{BB} \cdot q_{CB})^N}{(1 - (1 - w\bar{\bar{P}}_R) \cdot Q_{BB})N} \quad \text{for pretargeting},$$

$$(7.4) \qquad \bar{q}_{CB} = \frac{1 - (1 - (1 - w\bar{\bar{P}}_R) \cdot q_{CB})^N}{(1 - (1 - w\bar{\bar{P}}_R))N} \quad \text{for retargeting} .$$

The reason for including these additional factors in the denominator is that, on the average, the effectiveness of a weapon that survives all the damage and attrition barriers is higher than the calculated expected effectiveness of the weapon prior to the barriers. If there are SAM's, the kill probability for SAM's is entered in the same fashion as the kill probability for area defenses.

For missiles, $\bar{u}$ is computed in a similar fashion except the terms w and $(1 - \bar{\bar{P}})$ do not appear.

In determining N for value targets, it is clearly incorrect to assign the same number of warheads to defended and undefended targets.

If penetration aids are employed, it is assumed that a preferred mix of penetration aids and warheads has been adopted, and that the local defense kill probability reflects this assignment. It is slightly incorrect under this assumption to make the basic kill probability the same for a warhead whether attacking a defended or undefended target. However, this is a second-order effect.

Warheads are assigned to defended and undefended targets so as to equalize the probability of damage of the two types of targets. Thus, warheads are distributed according to the formula

$$(7.5)\quad n_2 = \max\left[\frac{M \cdot x_C}{C_D\left[\dfrac{\log(1-u)}{\log(1-\ell u)}\right] + C_u},\ 1\right] \quad \text{for undefended targets,}$$

$$(7.6)\quad n_1 = \frac{Mx_C - n_2 C_u}{C_D} \quad \text{for defended targets,}$$

where C_D is the number of defended cities, C_u is the number of undefended cities, and ℓ is the kill probability of local defenses.

Fallout effects are computed by introducing a table (see Table 4) relating the number of fallout casualties as a function of the kind of weapon and the target system.

Table 4

FORM OF FALLOUT TABLE

	Defenses	Bombers	Missiles	Value targets
Missiles				
Bombers				

Entries in the table would be based both on policy questions (whether or not ground bursts will be used against certain targets, whether city-avoidance will be observed) and physical questions (size of warhead, density of non-urban population in the vicinity of targets, etc.). Entries are expressed in city units—i.e., the fraction of a city unit of population that will be casualties for the given weapon-target combination. At the option of the planner, the fallout casualties can be added to the value-target damage or it can be separately computed. In either case the number of fallout casualties is separately recorded.

8. PAYOFF AND CRITERION

The knottiest problem in analyzing a central nuclear war is the payoff and criterion. Central nuclear war is not only highly nonzero sum but also the outcomes can include cases that are catastrophic to one or both sides. The theory of nonzero sum games for noncooperative situations is in an unsatisfactory state, and methods of dealing with catastrophic payoffs are extremely elementary.

The present form of STROP is based on a fairly simple point of view, namely that the primary function of strategic aerospace weapons is target destruction. Other possible functions—prewar deterrence, intrawar deterrence, back-up to threats, etc.—are derivative. Hence, the basic measure of the outcome of a nuclear exchange is target destruction on each side. Similarly, the measure of the value of aerospace weapons in a counterforce role is the amount of target damage saved. Both of these measures can refer either to actual destruction or to potential destruction. Thus, the value of withheld weapons can be measured in terms of the potential damage they can achieve.

STROP distinguishes between value targets and military targets. The value targets are called cities as a convenient term, but they could represent industry, or nonaerospace military targets such as troop concentrations, naval yards, etc. Lumping all of these varied kinds of targets under one heading produces headaches of aggregation, both value-wise and vulnerability-wise. However, for a rapid survey of the damage capabilities of forces, these difficulties arise for any technique of analysis. The basic payoff is, then, damage to value targets, either actual or potential. STROP evaluates potential damage by completing the interchange—i.e., by expending weapon against value targets. Interior to STROP there is no formal difference between actual and potential damage other than consideration of timing. The distinction is made in interpretation of the results.

Value targets are expressed in terms of target units. These are essentially elements that can be destroyed with reasonable probability with one weapon. A large city, or large industrial complex, would be represented as several target units. In the test runs, we used city units of about 200,000 population. Thus Moscow would be represented by about 25 such units. In distributing local defenses and AICBM's it is assumed that each defended unit is defended to the same level.

The basic payoff space is thus the two element vector, damage to Red and damage to Blue (see Fig. 5). The limits

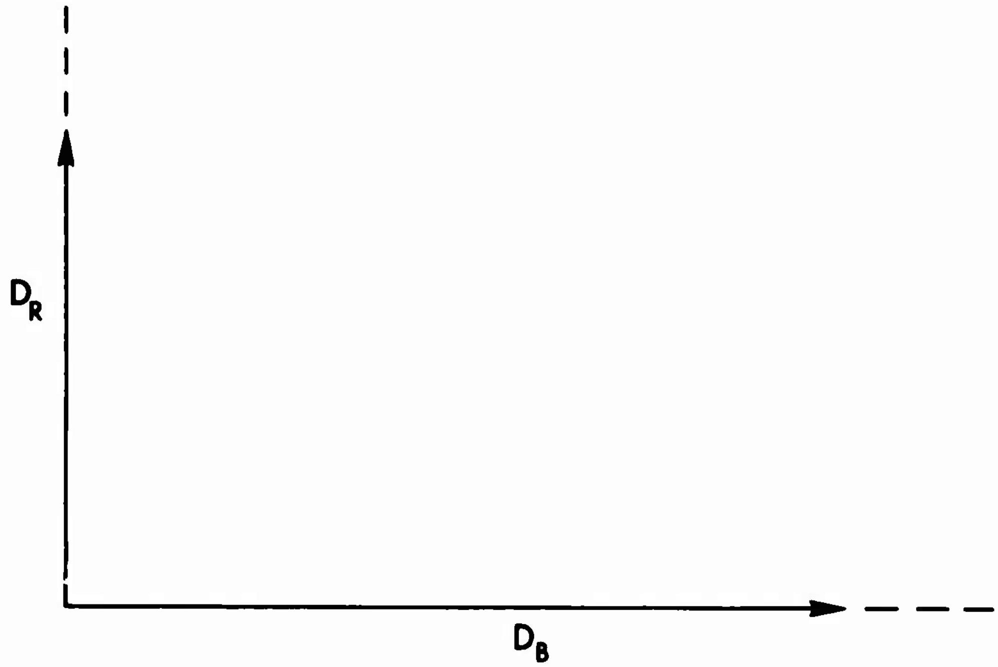


Fig. 5—Payoff space

on this space are vague; the size of the target system is clearly open to decision. Even if value targets are limited to population or industrial units, the question of where to stop can be reasonably raised.

One notion of some currency is that of <u>critical level</u>. This could be interpreted as the level of damage at which the nation is no longer viable, or the level at which it has lost the war irrespective of what happens to the other side, or the level ("unacceptable" level) at which no outcome would appear "worth it." All of these are vague and

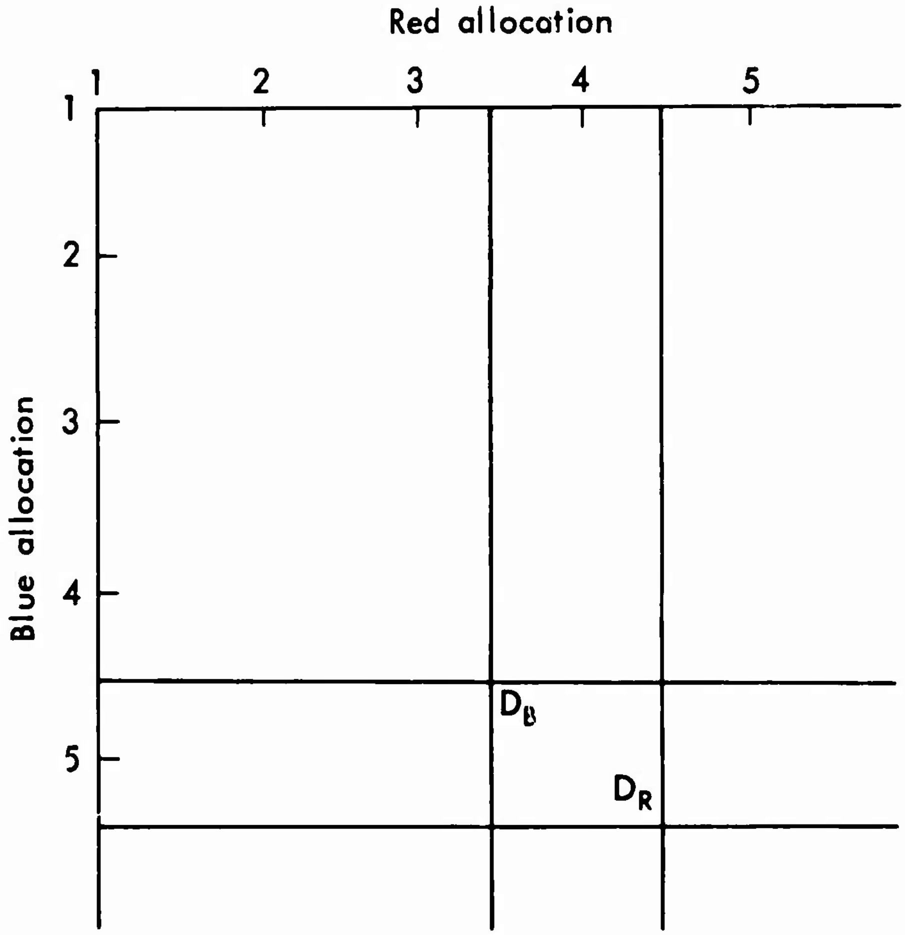
intuitive notions which are difficult to quantify. Nevertheless, some such notion lies behind phrases such as assured retaliation, and lies behind the selection of targets for a war plan.

The number of value targets is a pure input to STROP. For many purposes, it would probably be desirable to set these numbers sufficiently high so that some value targets remained no matter what level of attack is assumed. For other purposes it is clearly desirable to limit the size of the value-target system; e.g., questions of effectiveness of retargeting depend on the ratio of weapons to targets.

One further consideration relating to game-theoretic analysis is important. This has to do with the issue of mixed strategies. Especially for those cases where catastrophic outcomes are involved, mixed strategies appear inappropriate. Not only is there no way to compare catastrophic with noncatastrophic outcomes—and therefore no way to attach weights to strategies—but the sort of "Russian roulette" implied by the use of mixtures is foreign to military as well as civilian thinking. Hence, any decision procedure would presumably have to arrive at pure strategies even if these are not strictly optimal.

One criterion that appears reasonable from the military point of view is dominance. This is illustrated in Fig. 6 where the outcome matrix of a STROP run is shown schematically. Calling Blue allocations A_B, and Red A_R, for each pair (A_B, A_R) there is the corresponding pair of damages (D_B, D_R). A Blue allocation A_{Bi} is said to dominate another A_{Bj}, $A_{Bi} > A_{Bj}$, if and only if

$$\left.\begin{array}{l} D_{Bik} \leq D_{Bjk} \\ \\ D_{Rik} \geq D_{Rjk} \end{array}\right\} \quad \text{for all } k$$



Dominance: $$A_i > A_j \equiv \left.\begin{array}{l} D_{Bik} \le D_{Bjk} \\ D_{Rik} \ge D_{Rjk} \end{array}\right\} \text{ for all } k$$

at least one inequality strict

Fig. 6—STROP allocation analysis

and at least one of the inequalities is strict; in other words A_i dominates A_j if A_i results in more damage to Red and less to Blue, irrespective of what Red does. The converse condition defines dominance for Red.

Double dominance of the sort just defined is a very strong condition, and ordinarily would not be expected to apply to most allocations. However, as a matter of fact, in the test cases so far run—and there doesn't appear any reason to believe these are unique—the dominance criterion turns out to be very powerful. Most allocations on both sides are dominated out. Starting with a sample matrix of 168 x 168 (or 28,000 pairs), dominance reduced the matrix to roughly a 10 x 10. This is a reduction factor of about 300. The basic reasons for the power of simple dominance appear to be these: (1) Factors such as CEP, warhead size, and vulnerability of the target strongly determine whether a given weapon is efficiently employed against a given target system. (2) There are rapidly decreasing returns for additional weapons against a given target system. (3) Missions that depend on long chains of probabilistic events for their effectiveness (e.g., employment of missiles against missiles to limit damage) are inherently inefficient.

Although a reduction to a 10 x 10 is a very large simplification, it still is not the same as selecting preferred allocations on either side. There are several ways one could proceed. One way is to define the payoff as the difference between the two damages, i.e., $P_B = D_R - D_B$. This definition has the advantage that it makes the game zero-sum, and includes the desire on each side to maximize damage to the other side (actual or potential) and to minimize damage to itself. However, it imposes a very simple trade-off of targets. A somewhat less elementary payoff function would be to use a weighted difference, e.g.,

$$P_B = D_R - \alpha D_B, \; P_R = D_B - \beta D_R.$$

This payoff is already nonzero sum but does not include the element of catastrophic loss. It is only slightly more complex to include a factor which takes into account the desire of a given antagonist not to exceed his critical level.

This consideration is illustrated in Fig. 7. Presumably at some level of damage to Blue, labeled here the critical level, C_B, Blue has lost the war no matter what the status of Red, and similarly there is such a critical level for Red. At levels of low damage to both sides, it may be reasonable to assume that there is some form of trade-off of target for target. But as Blue's level of damage approaches that of the critical the relative value of his own targets over those of enemy targets increases, and an equal-value curve will then presumably become asymptotic to the line represented by the critical level [2].

A simple expression for such a payoff is indicated in Fig. 8. Here the simple difference in damage to value targets is modified by an expression containing a scaling factor A in the numerator, and in the denominator, the difference between Blue's damage and his critical damage. At lower levels of damage the expression is practically equivalent to the difference between the two damages. But at high levels of damage the correction factor takes over and as Blue's damage approaches the critical level it becomes negatively infinite. This is only one of a very large number of possible formulations which have the properties mentioned above. It does have the advantages of simplicity and of fairly direct interpretation. When this payoff is applied to the undominated matrix remaining and the matrix analyzed again using dominance (but in this case dominance on the new payoff), the matrix receives a further reduction, and in most cases it is reduced to a 1×1.

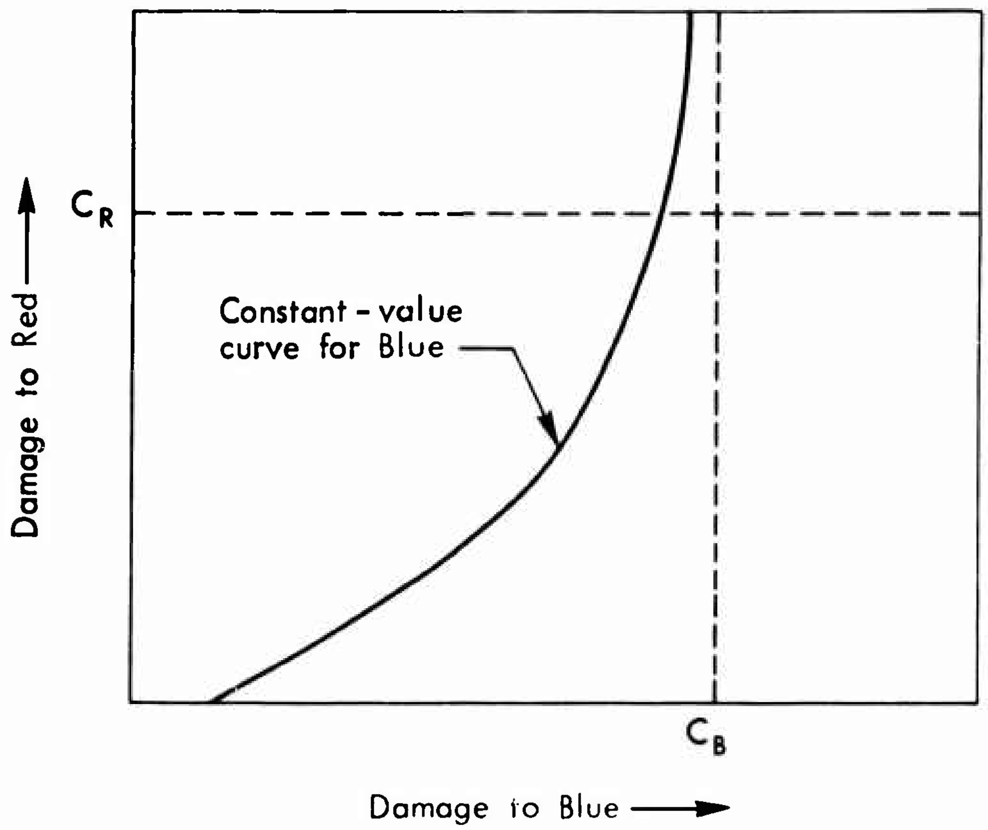


Fig.7—Assumption of increasing concern

$$P_B = D_R - D_B - \frac{A}{C_B - D_B} \cdot$$

$$P_R = D_B - D_R - \frac{B}{C_R - D_R} \cdot$$

	Red	Blue
Payoff	P_R	P_B
Damage	D_R	D_B
Critical level	C_R	C_B
Scaling factor	B	A

Fig.8—Simple form of assumption of increasing concern

The assumption of increasing concern can be expressed in a more direct fashion without the notion of critical levels simply by adding a factor to the payoff which is a rapidly decreasing function of the damage. The expressions

$$P_B = D_R - D_B - KD_B^2$$

$$P_R = D_B - D_R - LD_R^2$$

where K and L are constants, are a simple form of this more direct assumption. When this payoff is used in STROP, preferred strategies are selected which are either identical to or very similar to those selected with the assumption using critical levels. In addition the results are quite insensitive to changes in the constants K and L. Changes by a factor of 10 (e.g., from 2×10^{-3} to 2×10^{-2}) produced no change in the preferred strategies.

Since dominance with the increasing concern payoff generally reduces the matrix to a 1×1, the routine is in effect selecting a max-max point. This point can be searched for in a more direct fashion. A routine has been coded whereby the strategies are listed for each side in order of decreasing allocation of missiles to value-targets. For each Blue strategy in turn, then, Red strategies are explored until the Red payoff begins to decline, defining a maximum Red payoff for that Blue strategy. The exploration continues until the Blue payoff at the Red maximum begins to decline. This exploration in general will conclude well before all strategy pairs have been examined, substantially reducing the running time.

9. FORCE STRUCTURES EVALUATION

STROP can be used to evaluate force mixes as well as target allocations. The variant of STROP designed to evaluate force structures is called STROP II. A budget level can be input for each side, and the routine will generate a sample of additional forces (i.e., forces in addition to those already in inventory) whose costs add up to the given budget level. For each force structure pair (one for Red and one for Blue), the routine plays the target allocation game and selects preferred allocations for each side, as illustrated in Fig. 9. In this fashion a sample payoff matrix is generated, where the damages to Red and to Blue are those resulting from the preferred employments of the given force mixes. As with target allocations, dominance of either the pure type or the increasing concern type can be used to eliminate undesirable force mixes on each side. From the test runs we have made to date, dominance appears to be as powerful in eliminating ineffective force structures as it is in eliminating ineffective employments.

Different budget levels can be input for each side, as well as different force mix selection rules. Cost curves are input for each force element. There are seven elements in the present version of STROP II: bombers, missiles, bomber bases, fighter bases, local defenses, ABM's, and civil defense.

A typical cost curve might look like Fig. 10. This curve reflects the fact that a new missile system is being purchased; an initial outlay for research and development is required, and increasing returns to scale occur with larger purchases. If several different missile types are being bought at once, the curve need not be as simple

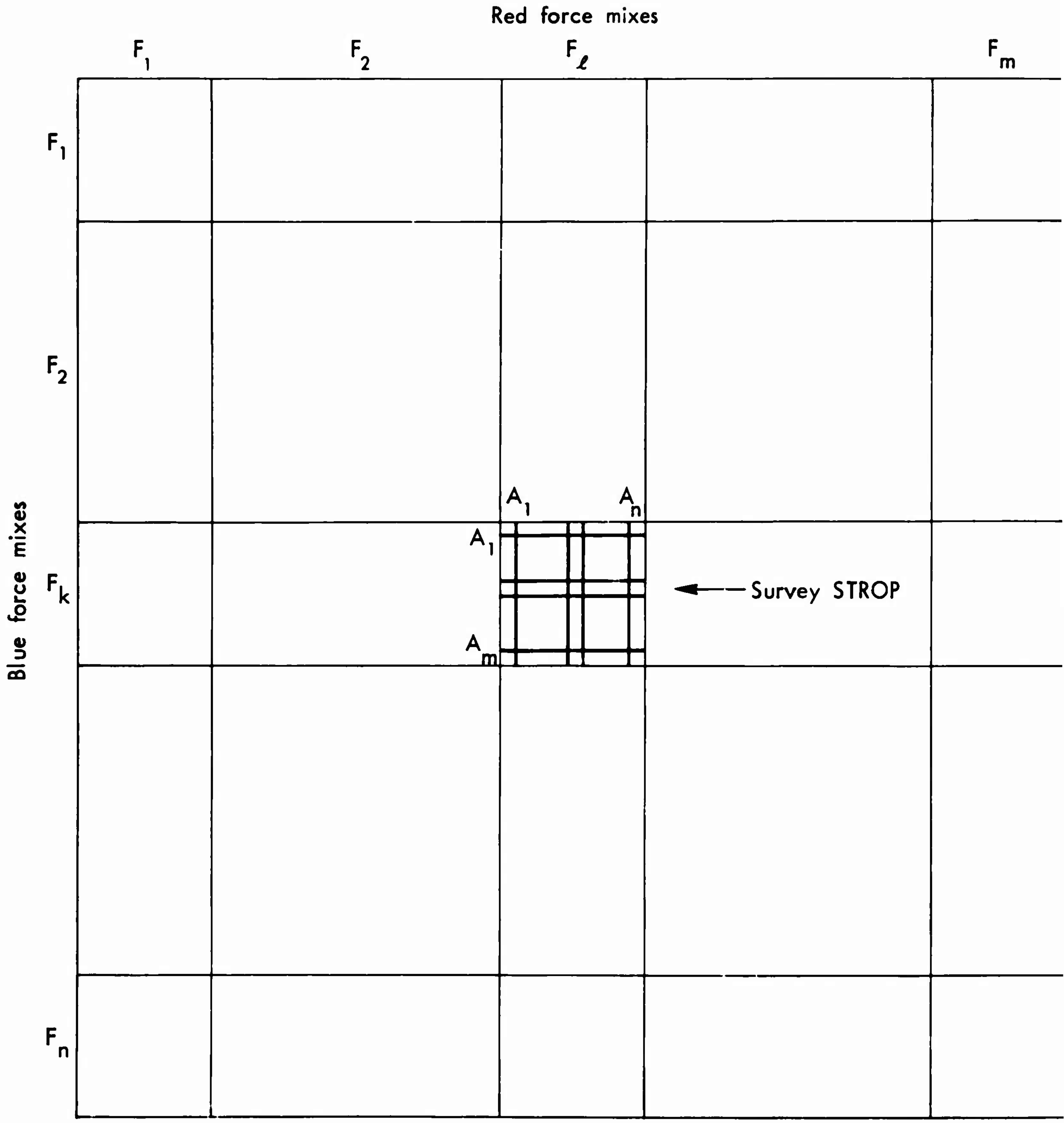


Fig. 9—STROP force-structures analysis

as in Fig. 10. In fact it need not even be monotonically increasing with dollars. For example, the planner might

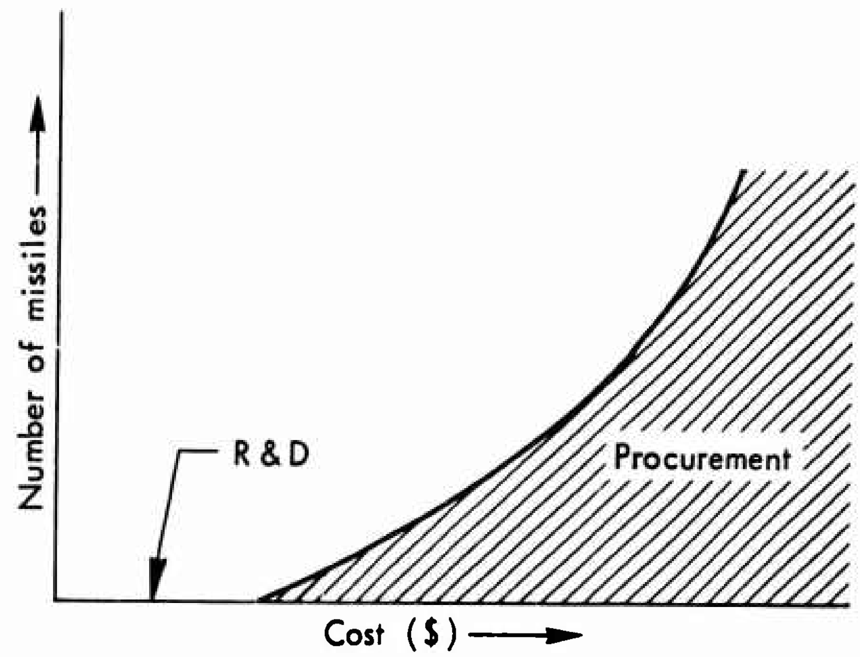


Fig. 10—Typical cost curve

have in mind simply buying additional missiles of the initial kind for low budget levels, but buying a new and more expensive missile at higher budget levels; in which case the number of missiles purchased might drop at the point of introducing the new system, as in Fig. 11.

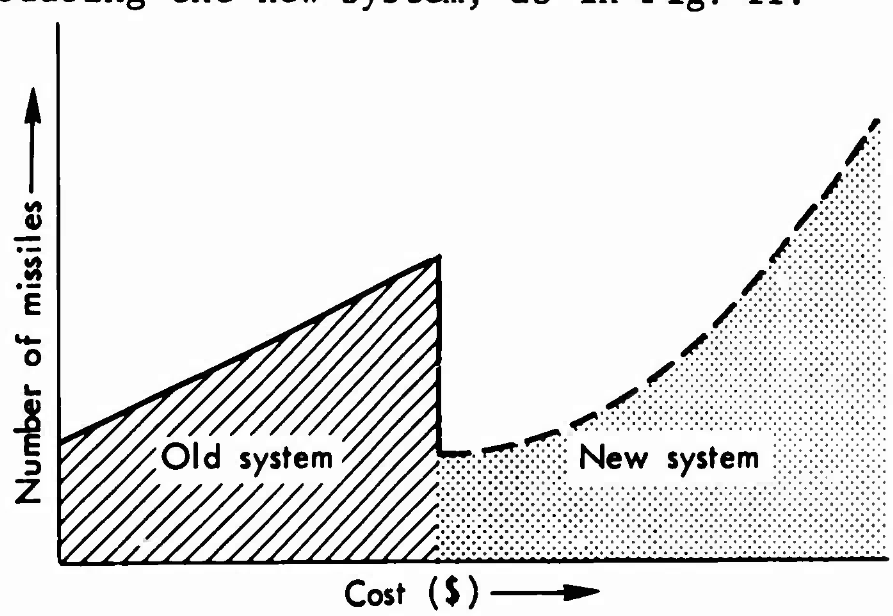


Fig. 11—Cost curve, new system

In addition to the cost-number curves, cost-performance curves must be input, to reflect the changing capabilities of new forces. Figure 12 is a typical curve for the kill probability of missiles on value targets, assuming that at some budget level the missiles will be retrofitted with multiple reentry vehicles, since u_C is the kill probability

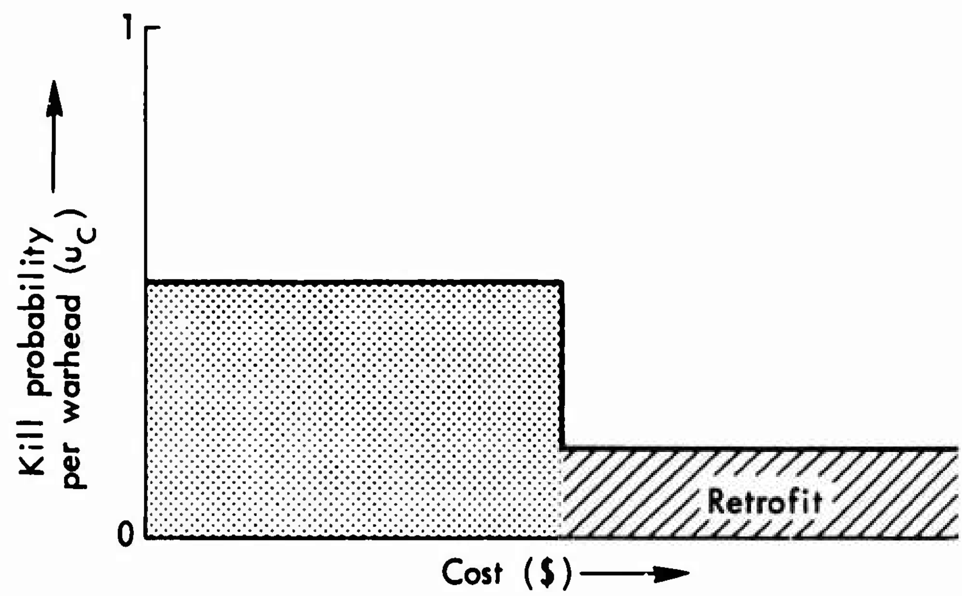


Fig. 12—Cost-capabilities curve

per warhead. Corresponding curves must be drawn for the kill probabilities of fighters, local defense, and AICBM's. The situation with regard to missile-on-missile effectiveness of a missile depends on the buying program of the enemy as well—e.g., he may harden his missiles.* This is handled by a pair of tables, which contain the Blue-on-Red and Red-on-Blue missile-missile effectiveness at various outlays for missiles on each side, as in Fig. 13. There is a corresponding table for Red.

*Strictly speaking this is true for all weapon-target combinations, but we have made the simplifying assumption that the hardness of bomber fields and fighter bases will not be changed. A small problem exists at present with regard to blast shelters. Strictly speaking, these should be handled like missile-missile combinations. At present this capability has not been programmed for STROP.

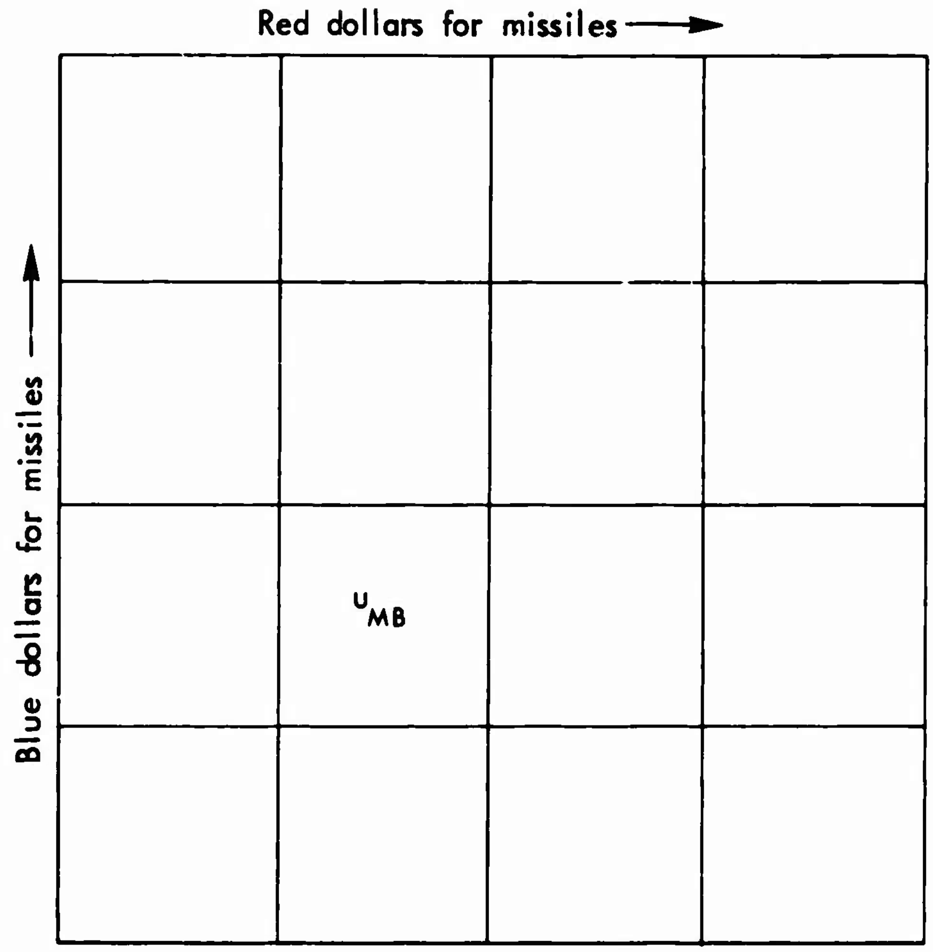


Fig. 13—Blue missile-on-missile effectiveness

Fallout shelters are defined by a level of protection index which is, in effect, the factor used to degrade the fallout effects computed from the fallout table described in Sec. 7.

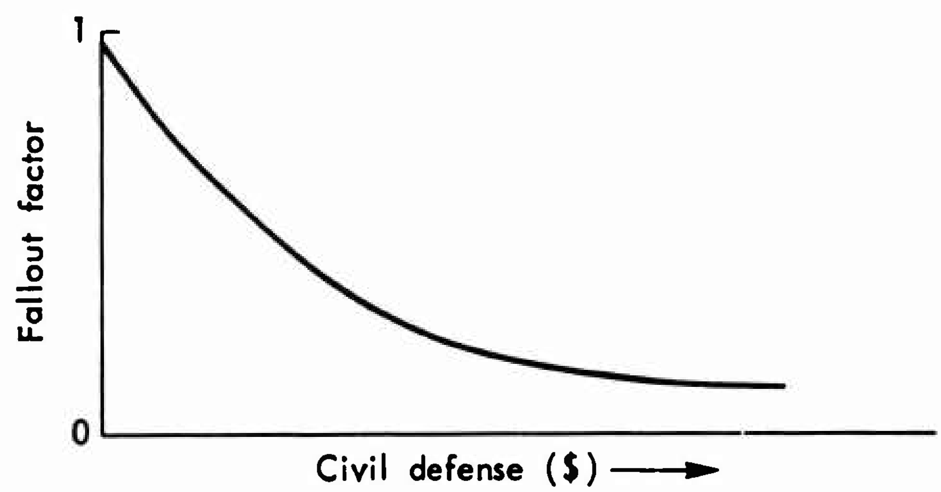


Fig. 14—Level of protection curve

From the attacker's point of view, fallout effects are not only a function of the budget level (e.g., they are dependent on warhead size) but also of policy (airburst vs. groundburst, city avoidance, etc.). The fallout table in STROP II does double duty. It acts as a policy formulation and also fixes the fallout effects of a nominal warhead for ground bursts. The fallout effects are modified by a factor which is a function of the budget level. Separate factors are tabled for bombers and missiles.

In general, numbers of additional forces are simply added to existing forces to produce the new mix. Capabilities, on the other hand, must be averaged. This is done by a simple weighted average where the weights are the numbers of existing and new forces, respectively. One area in which this is likely to produce some distortion is that of multiple warheads. A simple weighted average of the number of warheads may induce a different form of attack by the enemy than a mixed force consisting of some multiple warhead and some single warhead missiles (providing he knows which silos contain which missiles). However, there is no way to resolve this problem in the present highly aggregated version of STROP, and the outcomes must be evaluated with this limitation in mind.

The problem of surveying a wide range of force structures on both sides is several orders of magnitude larger than the problem of surveying a range of target allocations. In the first place, the number of elements is greater. Essentially, for target allocation there are four elements—three for missiles and one for bombers—since the allocations add up to one. For force structures there are six (with a fixed budget). Secondly, as noted above, for each pair of force structures, a preferred target allocation on each side must be computed. Finally, the scale of expenditure may be different for each force element; for example, if the total buy is set at 30 billion dollars for Blue, it appears

unlikely that any planner would want to spend the entire increment on, say, local defenses against bombers. Thus, local defenses might be acquired in 100 M$ steps, whereas ABM's might be acquired in 1 B$ steps. Since these steps may be incommensurable, the budget increment might have to be expressed, say, as 30 B$ ± 500 M$.

Several measures have been programmed into STROP II to reduce the size of the force-structures survey. The use of the direct form of the assumption of increasing concern payoff and the max-max computation for preferred strategies (p. 27a) leads to a reduction in running time by a factor of about 3.

In addition, a further reduction can be obtained by performing the bomber allocation analytically. Since in the present version of STROP bombers attack only defenses and value targets, the problem is to find some analytic way to prorate bombers between defenses and value targets.

The preferable criterion is to maximize the number of value targets destroyed by bombers. This criterion is difficult to apply, since value-target damage is a function of the distribution of bombers between locally defended value targets and nondefended targets. A simpler criterion is to maximize the number of bombers surviving area defenses and allocated to value targets. In the Appendix, a formula is derived which defines this maximum. Strictly speaking, the formula is valid only if the bomber forces can be retargeted after the enemy attack on bomber bases (since the preferred allocation depends on the surviving number of bombers).

Using the formula to allocate bombers reduces the sample size by a factor of 3 on each side or by a factor of 9 for the total sample, in the case of 20 percent allocation steps. It would require 2 to 4 hours to run a sample of 20 x 20 force structures, or 400 force-structure pairs. This is still somewhat expensive, considering the level of aggregation of the results, but appears to be within reason.

Appendix

BOMBER ALLOCATION BETWEEN DEFENSE AND VALUE TARGETS

We want to maximize the number of bombers available for attack on value targets after passing through the area defense barrier. Let B^* designate these bombers. We have

$$(A.1) \qquad B^* = (1 - \bar{\bar{P}}) \cdot \bar{B} \cdot y,$$

where $\bar{\bar{P}}$ is the area kill probability after attack by missiles and bombers, $\bar{B}$ is the number of bombers surviving ground attack, and y is the proportion of bombers allocated to value targets. We can write

$$\bar{\bar{P}} = \frac{\bar{\bar{D}}}{\bar{D}} \cdot \bar{P} = \frac{\bar{D} - (1 - \bar{P}) \cdot u_D \cdot (1 - y) \cdot \bar{B}}{\bar{D}} \cdot \bar{P}$$

$$(A.2) \qquad = \left[1 - (1 - P) \cdot u_D \cdot (1 - y) \cdot \frac{\bar{B}}{\bar{D}}\right] \cdot \bar{P}.$$

Whence

$$(A.3) \quad B^* = \left[1 - \left[1 - (1 - \bar{P}) \cdot u_D \cdot (1 - y) \frac{\bar{B}}{\bar{D}}\right] \cdot \bar{P}\right] \cdot \bar{B} \cdot y.$$

Taking the derivative of B^* with respect to y and simplifying we get

$$(A.4) \quad \frac{dB^*}{dy} = \bar{D} - \bar{P} \cdot \bar{D} + (1 - \bar{P}) \cdot u_D \cdot \bar{B} \cdot \bar{P} \cdot (1 - 2y).$$

Setting $dB^*/dy = 0$ and simplifying we obtain

$$(A.5) \qquad y = \frac{1}{2} + \frac{\bar{D}}{2\bar{B} \cdot \bar{P} \cdot u_D}.$$

From (A.5) we see that $y \geq 1/2$; i.e., no more than 1/2 of the bomber force would be allocated to attacking defenses.

(A.5) was derived on the assumption that the expected kill of defenses is linear in the number of bombers assigned to the defense-busting mission. This assumption is false as soon as the number of warheads assigned to defenses becomes greater than the number of defenses. STROP first computes y on the basis of (A.5) and then compares $\bar{B}(1 - y)$ with $\bar{D}$. If $\bar{B}(1 - y) \leq \bar{D}$, the computed y is selected. However, if $\bar{B}(1 - y) > \bar{D}$, a more complicated expression must be used, taking into account the fact that multiple warheads will be delivered on defense targets. In this case, we have

$$\text{(A.6)}\quad B^* = \bar{B} \cdot y\left[1 - (1 - (1 - \bar{P}) \cdot u_D)^{\frac{\bar{B}\cdot(1-y)}{\bar{D}}}\right] \cdot \bar{P}.$$

Setting $a = (1 - (1 - \bar{P})) \cdot u_D$, $v = \bar{B}/\bar{D}$ and simplifying,

$$\text{(A.7)}\quad \frac{dB^*}{dy} = \bar{B}\left[1 - a^{v(1-y)} \cdot \bar{P}(1 - \log a \cdot v \cdot y)\right],$$

and setting dB^*/dy to zero we have

$$\text{(A.8)}\quad a^{v(1-y)} \cdot \bar{P}(1 - \log a \cdot v \cdot y) = 1.$$

In the STROP routine, (A.8) is solved approximately by initially setting y equal to the value given by (A.5) and reducing it by .05 steps until the expression on the left of the equality exceeds 1. In some cases this will result in a value of y such that $(\bar{B}/\bar{D})y$ is less than 1 (less than 1 warhead per defense target); in which case y is reset to $\bar{D}/\bar{B}$.

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10. ABSTRACT

This small, rapidly computable model of central nuclear war was developed to help the analyst to evaluate quickly a large number of possible plans, each of which depends on the actions of the enemy. The game, coded for an IBM 7040-44, evaluates a pair of Red and Blue allocations of missiles and bombers to some combination of four target systems: missile sites, bomber fields, bomber defenses, and value targets. One pair of Red and Blue allocations can be evaluated in about 1/50 of a second. Although the model is basically static, time enters into STROP in determining the average number of bombers or missiles at risk during enemy attack; in assessing the effect of warning and relative execute times; and in assessing the effect of rate of fire, a capability that can also be used to take into account the results of withholding portions of the forces for later attack. The outcome of nuclear exchange can be evaluated if the attacks on both sides are completely preplanned, or if one or both sides have a retargeting capability to compensate for losses to enemy fire. STROP distinguishes between value targets and military targets. In addition to exploring strategic plans, it can be used to evaluate force mixes.

11. KEY WORDS

Models
Military planning
Strategic warfare
Nuclear warfare
War gaming
Force structure
Allocation process